Appendix: Dispersion Equations & Paraxial Optics Formulae

The following equations may be used to determine the on-axis refractive index N_0 (λ) and the gradient constant $\sqrt{A}$ (λ) for all standard SELFOC® MicroLenses. The equations are identified by the lens type followed by the lens diameter in millimeters. These formulae are valid for wavelengths greater than 550 nanometers.

On-Axis Refractive Index: λ in μm

SLS:

$$N_0(\lambda) = 1.5477 + \frac{6.37\text{x}10^{-3}}{\lambda^2}$$

SLW:

$$N_0(\lambda) = 1.5868 + \frac{8.14\text{x}10^{-3}}{\lambda^2}$$

SLW-3.0, SLW-4.0:

$$N_0(\lambda) = 1.6107 + \frac{9.8\text{x}10^{-3}}{\lambda^2}$$

SLH:

$$N_0(\lambda) = 1.6294 + \frac{1.12\text{x}10^{-2}}{\lambda^2}$$

All of the common paraxial distances for SELFOC MicroLenses may be derived from the following ray-trace matrices. Variables r_1 and θ_1 are the input ray height and slope respectively, in a medium of index n_1 (see Figure A1). At the rear surface of the lens, r_2 and θ_2 are the corresponding output variables in a medium of index n_2. For the plano-convex lens, the curvature is defined as $C=(N_0 - n_1)/(N_0 R)$, where R is the radius of curvature.

Paraxial Ray-tracing Matrices:

plano-plano lens:

$$\begin{bmatrix} r_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \cos(Z\sqrt{A}) & \frac{n_1}{N_0\sqrt{A}} \sin(Z\sqrt{A}) \\ -\frac{N_0\sqrt{A}}{n_2} \sin(Z\sqrt{A}) & \frac{n_1}{n_2} \cos(Z\sqrt{A}) \end{bmatrix} \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$$

plano-convex lens:

$$\begin{bmatrix} r_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \cos(Z\sqrt{A}) - \frac{C}{\sqrt{A}} \sin(Z\sqrt{A}) & \frac{n_1}{N_0\sqrt{A}} \sin(Z\sqrt{A}) \\ -\frac{N_0}{n_2}\left|\sqrt{A} \sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right| & \frac{n_1}{n_2} \cos(Z\sqrt{A}) \end{bmatrix} \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$$

Index Gradient Constant: λ in μm, $\sqrt{A}$ in mm^{-1}

SLS-1.0:

$$\sqrt{A}(\lambda) = 0.4785 + \frac{7.157\text{x}10^{-3}}{\lambda^2} + \frac{3.749\text{x}10^{-4}}{\lambda^4}$$

SLS-2.0:

$$\sqrt{A}(\lambda) = 0.2339 + \frac{7.643\text{x}10^{-3}}{\lambda^2} - \frac{9.757\text{x}10^{-4}}{\lambda^4}$$

SLW-1.0:

$$\sqrt{A}(\lambda) = 0.5945 + \frac{3.936\text{x}10^{-3}}{\lambda^2} + \frac{5.539\text{x}10^{-4}}{\lambda^4}$$

SLW-1.8:

$$\sqrt{A}(\lambda) = 0.3238 + \frac{5.364\text{x}10^{-3}}{\lambda^2} + \frac{2.626\text{x}10^{-4}}{\lambda^4}$$

SLW-2.0:

$$\sqrt{A}(\lambda) = 0.2931 + \frac{2.369\text{x}10^{-3}}{\lambda^2} + \frac{7.681\text{x}10^{-4}}{\lambda^4}$$

SLW-3.0:

$$\sqrt{A}(\lambda) = 0.1973 + \frac{3.723\text{x}10^{-3}}{\lambda^2} + \frac{2.050\text{x}10^{-5}}{\lambda^4}$$

SLW-4.0:

$$\sqrt{A}(\lambda) = 0.1468 + \frac{2.654\text{x}10^{-3}}{\lambda^2} + \frac{3.960\text{x}10^{-6}}{\lambda^4}$$

SLH-1.8:

$$\sqrt{A}(\lambda) = 0.4151 + \frac{4.137\text{x}10^{-3}}{\lambda^2} + \frac{7.652\text{x}10^{-4}}{\lambda^4}$$

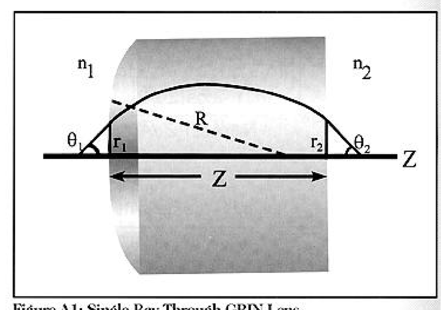

Figure A1: Single Ray Through GRIN Lens

Figure A2 illustrates the relative positions of the cardinal points for a typical SELFOC® Lens. Distances measured to the left of a reference point are negative; to the right, positive.

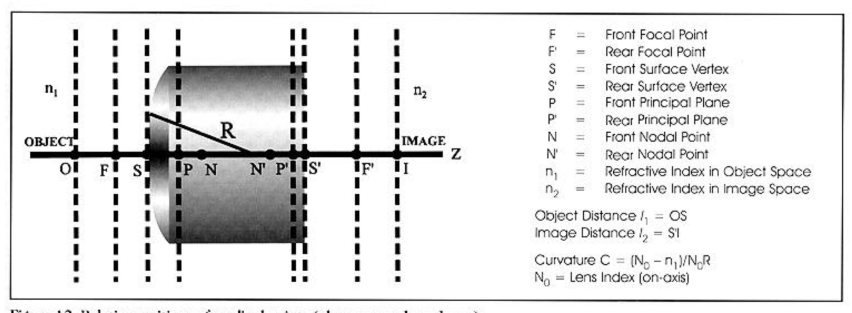

Figure A2: Relative positions of cardinal points (plano-convex lens shown)

Paraxial Distances for Plano-Plano Lenses:

front focal length:

$$\overline{FS} = \frac{n_1 \cos(Z\sqrt{A})}{N_0\sqrt{A}\sin(Z\sqrt{A})}$$

effective front focal length:

$$\overline{FP} = \frac{n_1}{N_0\sqrt{A}\sin(Z\sqrt{A})}$$

rear focal length:

$$\overline{S'F'} = \frac{n_2 \cos(Z\sqrt{A})}{N_0\sqrt{A}\sin(Z\sqrt{A})}$$

effective rear focal length:

$$\overline{P'F'} = \frac{n_2}{N_0\sqrt{A}\sin(Z\sqrt{A})}$$

front principal distance:

$$\overline{SP} = \frac{n_1\left[1 - \cos(Z\sqrt{A})\right]}{N_0\sqrt{A}\sin(Z\sqrt{A})}$$

rear principal distance:

$$\overline{S'P'} = \frac{-n_2\left[1 - \cos(Z\sqrt{A})\right]}{N_0\sqrt{A}\sin(Z\sqrt{A})}$$

front nodal distance:

$$\overline{SN} = \frac{n_2 - n_1 \cos(Z\sqrt{A})}{N_0\sqrt{A}\sin(Z\sqrt{A})}$$

rear nodal distance:

$$\overline{S'N'} = \frac{-n_1 + n_2 \cos(Z\sqrt{A})}{N_0\sqrt{A}\sin(Z\sqrt{A})}$$

image distance:

$$L_2 = \overline{S'I} = \frac{-(n_1 n_2/\sqrt{A})\sin(Z\sqrt{A}) - n_2 N_0 L_1 \cos(Z\sqrt{A})}{n_1 N_0 \cos(Z\sqrt{A}) - N_0^2 L_1\sqrt{A}\sin(Z\sqrt{A})}$$

transverse magnification:

$$M_T = \frac{n_1}{n_1\cos(Z\sqrt{A}) - N_0 L_1\sqrt{A}\sin(Z\sqrt{A})}$$

longitudinal magnification:

$$M_L = \frac{n_1 n_2}{\left[n_1\cos(Z\sqrt{A}) - N_0 L_1\sqrt{A}\sin(Z\sqrt{A})\right]^2}$$

angular magnification:

$$M_A = \frac{n_1\cos(Z\sqrt{A}) - N_0 L_1\sqrt{A}\sin(Z\sqrt{A})}{n_2}$$

Paraxial Distances For Plano-Convex Lenses:

front focal length:

$$\overline{FS} = \frac{n_1 \cos(Z\sqrt{A})}{N_0\left[\sqrt{A}\sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right]}$$

front principal distance:

$$\overline{SP} = \frac{n_1\left[1 - \cos(Z\sqrt{A})\right]}{N_0\left[\sqrt{A}\sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right]}$$

effective front focal length:

$$\overline{FP} = \frac{n_1}{N_0\left[\sqrt{A}\sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right]}$$

rear principal distance:

$$\overline{S'P'} = \frac{-n_2\left[1 - \cos(Z\sqrt{A}) + (C/\sqrt{A})\sin(Z\sqrt{A})\right]}{N_0\left[\sqrt{A}\sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right]}$$

rear focal length:

$$\overline{S'F'} = \frac{n_2\left[\cos(Z\sqrt{A}) - (C/\sqrt{A})\sin(Z\sqrt{A})\right]}{N_0\left[\sqrt{A}\sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right]}$$

front nodal distance:

$$\overline{SN} = \frac{n_2 - n_1\cos(Z\sqrt{A})}{N_0\left[\sqrt{A}\sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right]}$$

effective rear focal length:

$$\overline{P'F'} = \frac{n_2}{N_0\left[\sqrt{A}\sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right]}$$

rear nodal distance:

$$\overline{S'N'} = \frac{-n_1 + n_2\left[\cos(Z\sqrt{A}) - (C/\sqrt{A})\sin(Z\sqrt{A})\right]}{N_0\left[\sqrt{A}\sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right]}$$

image distance:

$$L_2 = \overline{S'I} = \frac{-(n_1 n_2/\sqrt{A})\sin(Z\sqrt{A}) - n_2 N_0 L_1\left[\cos(Z\sqrt{A}) - (C/\sqrt{A})\sin(Z\sqrt{A})\right]}{n_1 N_0\cos(Z\sqrt{A}) - N_0^2 L_1\left[\sqrt{A}\sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right]}$$

transverse magnification:

$$M_T = \frac{n_1}{n_1\cos(Z\sqrt{A}) - N_0 L_1\left[\sqrt{A}\sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right]}$$

longitudinal magnification:

$$M_L = -\frac{n_2}{n_1}\left[\cos(Z\sqrt{A}) - \frac{C}{\sqrt{A}}\sin(Z\sqrt{A}) - \frac{N_0 L_2}{n_2}\left[\sqrt{A}\sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right]\right]$$

angular magnification:

$$M_A = \frac{n_1}{n_2}\cos(Z\sqrt{A}) - \frac{N_0 L_1}{n_2}\left[\sqrt{A}\sin(Z\sqrt{A}) + C\cos(Z\sqrt{A})\right]$$